STABILITY OF A PERTURBED CONDUCTING GAS FLOW IN A MAGNETIC FIELD FOR ARBITRARY MAGNETIC REYNOLDS NUMBERS

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UDC 537.538.4

A numerical calculation is made which describes the conversion into a T-layer of a finite perturbation in electrical conductivity imposed on a one-dimensional supersonic flow of a compressible medium for a finite value of the magnetic Reynolds number. The development of the injected perturbation is significantly affected by the magnetic Reynolds number of the unperturbed flow, and to each value of this number there corresponds a particular boundary region in which the perturbation is "taken up" by the magnetic field into an induced T-layer. The stability is investigated in the linear approximation for a minimal perturbation, and the dispersion equation is solved with allowance for gradients in the unperturbed parameters. It is shown that an overheating instability can arise in the system and lead to the formation of a T-layer.

The first calculations on the development of a perturbation into a T-layer (take-up) in a flow of weakly conducting gas, i.e., for an unperturbed magnetic Reynolds number

$$R_m{}^+ = 4\pi\sigma_1 U_1 l \,/\, c^2 \tag{1}$$

much less than unity, were made in [1]. In Eq. (1), σ_1 and U_1 are the electrical conductivity and flow velocity at the duct input and l is the length of the duct. It was found that the formation of a T-layer occurs if the MHD interaction parameter S, corresponding to the parameters of the injected perturbation and equal to $S = R_m R_H$, is greater than some critical value S^+. [Here R_m is the magnetic Reynolds number, defined in contrast to (1) over the average parameters in the perturbation and over its length, and $R_H = H^2/8\pi P$ is the ratio of the magnetic field pressure to the gasdynamic pressure of the gas.]

We are interested here in the stability of a flow of highly conducting gas in a magnetic field with $R_m^+ > 1$ against a temperature perturbation of finite and then of infinitesimally small amplitude. We consider the one-dimensional flow of a conducting, compressible fluid in a magnetic field inside a cylindrical duct.

A numerical solution of the nonstationary equations of magnetohydrodynamics is obtained from the results in [2], and allowance is made for the singularities used in [1].

We consider the supersonic flow of a gas consisting of a mixture of dissociated hydrogen and a 1% ionizing addition of lithium. The flow expands along the radius of the channel perpendicular to the magnetic field lines. The parameters at the input to the duct (the stagnation temperature T_0 and the stagnation pressure P_0) are chosen such that the hydrogen is almost completely dissociated. Because the amount of lithium is small, the gas can be considered as ideal [3] over the range of parameters used.

The electrical conductivity $\sigma(\rho, T)$ is calculated from the equation for a partially ionized gas [4]. The necessary value of the effective collision cross section for electrons with neutral hydrogen atoms Q_{ea} was taken from [5].

The following boundary conditions were used in integrating the system of equations. The gasdynamic parameters (temperature T_1, density ρ_1, velocity U_1 at the left boundary) are taken to be constant and are

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Teknicheskoi Fiziki, No. 3, pp. 3-9, May-June, 1973. Original article submitted June 27, 1972.

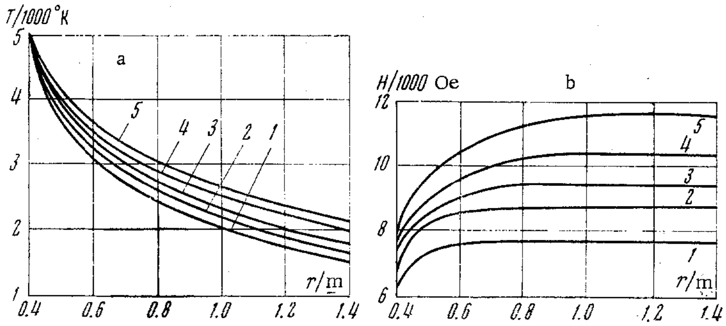

Fig. 1

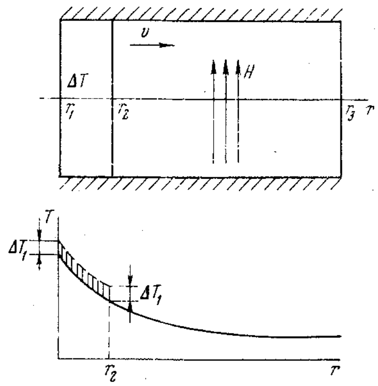

Fig. 2

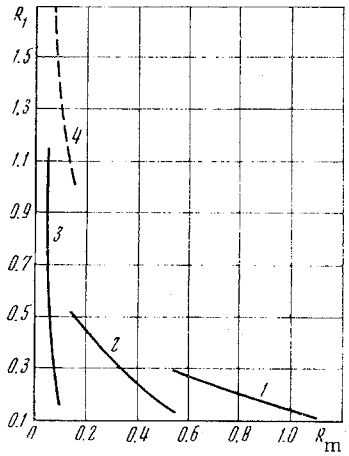

Fig. 3

determined from the stagnation parameters T_0 and P_0 at the input to the duct. For the condition on the magnetic field at the left boundary, it is assumed that for $r < r_1$ this field does not interact with the gas, i.e., it is constant in this region. From the integral Maxwell equations and Ohm's law, we then obtain the boundary condition at $r = r_1$ for the magnetic induction equation. The field at the right boundary is kept constant and equal to its initial value.

We assume that at the initial instant of time $t = 0$ there is a flow with a known parameter distribution inside the duct:

$$T = T(r), \quad \rho = \rho(r), \quad U = U(r), \quad H = H(r)$$

This distribution was found from a numerical solution of the system of ordinary differential equations of magnetohydrodynamics describing a one-dimensional stationary gas flow in a duct. The set of stationary sources for various values of the magnetic field [Fig. 1 shows the stationary distributions of temperature (a) and magnetic field (b) along a radius] was then used to provide the initial data for the calculation of the nonstationary problem. In Fig. 1a, curves 1-5 correspond to $H/1000 = 7.75$, 8.79, 9.45, 10.366, and 11.65 Oe.

A local perturbation in temperature ΔT_1 is given in the region $r_1 < r < r_2$ (Fig. 2). On the one hand, the subsequent evolution of this perturbation [1] is determined by its interaction with the magnetic field and the production of Joule heating, and on the other, the cylindrical expansion and the hydrodynamic dispersion decrease the temperature and, hence, the electrical conductivity.

If the interaction with the magnetic field plays the leading role, the perturbation is "taken up" and develops into a T-layer.

It can be shown that thermal conductivity and radiation do not affect the development of the instability for the given gas parameters since the amount of heat transferred from the perturbation by conductivity or radiation in one oscillation period is much smaller than the perturbation energy. Estimates on the basis of the gas parameter values used show that it is possible to neglect the effect of thermal transport processes on the value of the critical interaction parameter S^+ provided that the temperature and the length of the perturbation satisfy the conditions $T \leq 10^4$ °K and $\lambda \geq 1$ mm.

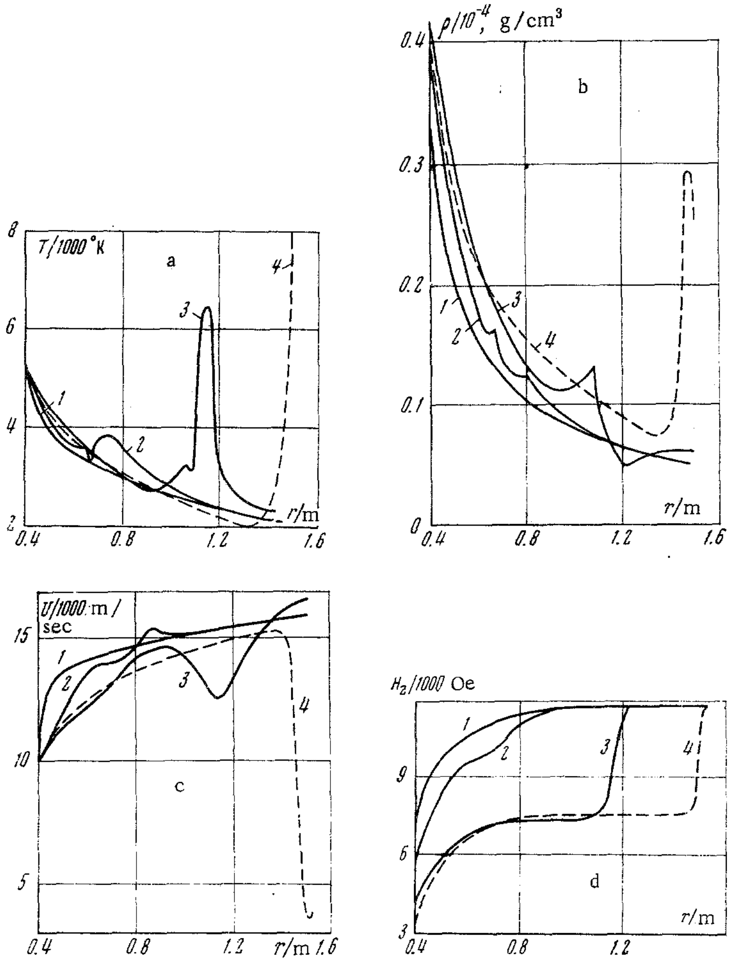

Fig. 4

In addition to a stationary source in the input region of the duct $r_1 < r < r_2$ with a size of ~ 0.2 times the total length of the duct $l = 1.1$ m, a finite temperature perturbation was imposed in the calculations. At the initial instant of time $t = 0$, an identical rise in temperature ΔT_1 is assumed for all points of the perturbation as shown for an arbitrary temperature distribution in Fig. 2. The velocity and density retain their former values.

The behavior of this local perturbation introduced into gas flows of various conductivities situated in a magnetic field was then investigated at subsequent moments of time. The stagnation pressure was $P_0 =$ 50 atm and the flows were distinguished by stagnation temperatures at the input of $T_0 = 8000$, 7000, and 6000°K. These values corresponded to $R_m{}^+ = 4.78$, 2.18, and 1. In each case different values were taken for the initial increase in temperature ΔT_1 and the perturbation length λ (i.e., we varied the value of R_m calculated over the length and temperature of the perturbation) for various values of the magnetic field obtained from the calculation of a stationary source. Thus, for example, with $R_m{}^+ = 4.78$, and magnetic field H in the range 6092-11,650 Oe, the parameter ΔT_1 was varied between 130-750°K corresponding to $R_m =$ 0.634-1.22. For each magnetic field and $R_m{}^+$ it is possible to find some critical value R_m above which the injected local temperature perturbation develops into a T-layer.

The results obtained for $R_m{}^+ = 4.78$, 2.18, and 1 are most clearly represented in the form of three boundary curves (curves 1, 2, and 3 in Fig. 3) constructed in the dimensionless perturbation coordinates $R_m - R_H$. Curve 4 corresponds to a value $R_m{}^+ \ll 1$. Here $R_H = H^2/8\pi P_+$ (H is the applied magnetic field and P_+ is the pressure averaged over the perturbation). The interaction parameter $S = S^+$ along these boundary curves. A perturbation develops into a T-layer if its parameters R_m and R_H define a point lying above the boundary curve $S = S^+$ and is attenuated if this point lies below the curve.

The position of the curves for $R_m{}^+ \geq 1$ can be explained by the different degree of interaction between the main flow and the magnetic field. For one and the same value of the applied magnetic field H (or R_H),

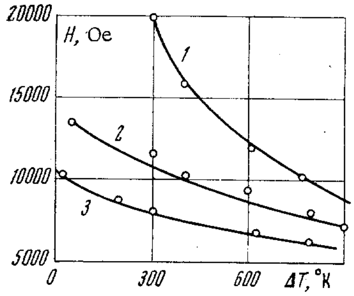

Fig. 5

a greater R_m^+ will correspond to a greater expulsion of the magnetic field at the duct input with a stationary flow. Thus, in the calculation of the stability of this stationary flow, the actual value of the magnetic field $H_+ = H - H_u$ (H_u is the magnetic field from the currents in the plasma) is smaller and for the perturbation to be taken up for a given H_+, the value of R_m in the perturbation must be increased.

For comparison with the results on the induction of a T-layer in a flow of a weakly conducting gas [1], Fig. 3 shows a single curve for all gas flows characterized by $R_m^+ \ll 1$ (dashed line). This curve lies above all the boundary curves we have considered here. This means that when $R_m^+ \ll 1$, a greater value of the interaction parameter S^+ is required for the perturbation to be taken up. In a weakly conducting gas it is characteristic to have a unilateral effect from the perturbation on to the gas flow, but when $R_m^+ > 1$ the flow in which the perturbation occurs does in fact affect the development of the perturbation. The electric currents surrounding the plasma add to the currents in the perturbation and the interaction with the magnetic field is improved; for a particular perturbation characterized by R_m, a smaller value of S^+ is required in a highly conducting gas than in a gas flow for which $R_m^+ \ll 1$.

Another important result from this analysis of flow stability is that for every $R_m^+ > 1$ there is a certain high value of critical magnetic field H^* such that when $H \geq H^*$ it is impossible to get a stationary flow as $t \to \infty$, even though no perturbation is injected [6]. For $R_m^+ = 4.78$, the critical value $H^* = 10{,}366$ Oe and for $R_m^+ = 2.18$ and 1 $H^* = 13{,}700$ and 20,000 Oe, respectively. The parameter distribution then corresponds to the development of a T-layer. This is evidence of the fact that the presence of dissipative processes caused by Joule heating and the growing dependence of the conductivity on temperature lead, even in the absence of a finite temperature perturbation $\Delta T_1 = 0$ ($R_m = 0$), not to the attenuation of small perturbations, but to their amplification and to a qualitative restructuring of the entire flow. This is illustrated in Fig. 4a–d by a T-layer which is gradually developing with time for $R_m^+ = 4.78$ and $H = 11{,}650$ Oe $> H^*$. Curves 1, 2, 3, and 4 refer to $t = 0.2 \cdot 10^{-5}$, $5 \cdot 10^{-5}$, and 10^{-4} sec.†

In addition to considering the stability of a stationary flow against finite-amplitude perturbations, we have also studied the reaction of the original state to infinitely small perturbations (which can be produced by a wide variety of sources). An overheating instability can be produced by the fact that the conductivity depends strongly on the thermodynamic flow parameters. The unperturbed parameters depend on the radius and so in linearizing the initial system of equations we include gradients in the unperturbed parameters (this was not done in [7–9]). We seek the solution of the linearized system in the form

$$\exp i(K_x x + K_y y - \omega t) \tag{2}$$

where K is the wave vector and ω the frequency of the oscillations.

Using (2), we get from the initial system the following dispersion equation:

$$D(\omega, K_x, K_y) = |a_{ij}| = 0 \tag{3}$$

$$a_{11} = U_0' - i\omega, \quad a_{12} = \rho_0' + i\rho_0 K_x$$

$$a_{13} = i\rho_0 K_y, \ a_{21} = U_0 U_0' + iK_x P_0 / \rho_0$$

$$a_{22} = \rho_0 U_0' + i\omega\rho_0, \quad a_{24} = iK_x P_0 / T_0$$

$$a_{25} = -j_0 + iB_0 K_x / \mu, \ a_{31} = iK_y P_0 / \rho_0$$

$$a_{33} = -i\omega\rho_0, \quad a_{34} = iK_y P_0 / \rho_0, \ a_{35} = iK_y B_0 / \mu$$

$$a_{41} = C_{v0} U_0 T_0' + P_0 U_0' / \rho_0, \ a_{43} = iP_0 K_y$$

$$a_{42} = P_0 C_{v0} T_0' + iK_x P_0, \ a_{44} = K^2 \lambda_* - i\rho_0 C_{v0}\omega$$

$$a_{45} = -2iK_x B_0' / \sigma_0 \mu^2, \ a_{52} = iK_x B_0 + B_0'$$

$$a_{53} = iK_y B_0, \ a_{54} = \beta B_0'' / \mu\sigma_0 T_0$$

$$a_{55} = -i\omega + U_0' + K^2 / \mu\sigma_0, \quad a_{14} = a_{15} = a_{23} = a_{32} = a_{51} = 0,$$

$$\beta = \partial \ln \sigma_0 / \partial \ln T_0$$

$$K^2 \lambda_* = (\beta / \mu^2 \sigma_0 T_0)(B_0')^2 + P_0 U_0' / \rho_0$$

†As in Russian orginal. Only three values are given for the four curves — Publisher.

The subscript 0 refers to the unperturbed flow parameters and the dashes denote differentials of the unperturbed quantities with respect to the radius; j_0 is the electric current density, B_0 is the magnetic induction, μ is the magnetic permeability, and C_{V0} is the thermal capacity.

This dispersion equation is a fifth-order polynomial in ω. A necessary condition for the appearance of instability is that

$$\mathrm{Im}\,\omega = \omega_i > 0 \tag{4}$$

for at least one of the frequencies determined from (3) for real values of K (the wavelength $\lambda \sim 1/K$ is given as equal to the counting interval in the problem of the stability of the original flow against finite perturbations). The inequality (4) is a necessary condition; a sufficient condition is that the amplitude of the perturbation should have sufficient time to grow while it is inside the duct. This last condition can be written

$$\omega_i \tau > 1 \tag{5}$$

Here τ is a characteristic time, equal in order of magnitude to $\tau = l/V$, where V is the phase velocity. Substituting into (5) the expression for the growth rate of an overheating instability, we get

$$\omega_i \tau \sim \frac{\partial \ln \sigma_0}{\partial \ln T_0} j_0^2 \tau / \sigma_0 \rho_0 C_{v0} T_0 = (\sigma_0 U_0^2 B_0^2 / \rho_0 C_{v0} T_0) \frac{l}{V} = R_m R_H > \frac{1}{2} \frac{\partial \ln T_0}{\partial \ln \sigma_0} \frac{V\lambda}{U_0 l} \tag{6}$$

Expression (6) can be rewritten

$$S = R_m R_H > S^+$$

In order to find the limits of stability, we have solved the dispersion equation (3) numerically for several points in a duct with $K_y = 0$. The most dangerous point (maximum ω_i) is at the duct input where the parameter gradients are large. The critical values obtained for the magnetic field are shown in Fig. 5 and agree with the results of the analysis of the stability against finite perturbations as the perturbation amplitude tends to zero ($\Delta T_1 \to 0$). The calculation of the critical field values was carried out by E. V. Kudryatseva.

Figure 5 also shows the variation of the perturbation amplitude ΔT_1 with the magnetic field for $R_m^+ =$ 1, 2.18, and 4.78 (curves 1, 2, and 3, respectively).

From our analysis we can conclude that the overheating instability can undergo subsequent nonlinear growth and become transformed to the state which has been called [1] a T-layer.

The authors thank L. M. Degtyarev, L. A. Zaklyaz'minskii, and A. P. Favorskii for useful discussions and advice during the completion of this work.

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